Calibration of Bond Coefficient for Deflection Control of FRP RC Members

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ABSTRACT: In 2006 the Italian Research Council (CNR) published a "Guide for the Design and Construction of Concrete Structures Reinforced with FRP Bars" (CNR-DT 203/2006); the approach followed is that of the limit states semi-probabilistic method; the model adopted for the design moment resistance of FRP RC sections is based on the design rules of Eurocode 2: it shall satisfy both strength and serviceability requirements. The present work developed in this context assesses the formulation suggested by the CNR-DT 203 for computing deflections of FRP RC members depending on FRP bars-to-concrete bond, by taking into account the bond coefficient "*m*". A calibration analysis was conducted in order to determine an optimum value for "*m*", based on a large experimental database available in literature, made of FRP RC elements subjected to four-points bending tests.

1 INTRODUCTION

According to CNR-DT 203/2006 and in compliance with the Eurocode 2 (2004), the computation of deflections of FRP reinforced concrete (RC) members can be performed by integration of the curvature diagram. Such diagram can be computed with non-linear analyses by taking into account both cracking and tension stiffening of concrete. Alternatively, simplified analyses are possible, similar to those used for traditional RC members. Experimental tests have shown that the model proposed by Eurocode 2 when using traditional RC members can be deemed suitable for FRP RC elements too. Therefore, the following equation to compute the deflection f can be considered:

$$f = f_1 \cdot \beta_1 \cdot \beta_2 \cdot \left(\frac{M_{cr}}{M_{max}}\right)^m + f_2 \cdot \left[1 - \beta_1 \cdot \beta_2 \cdot \left(\frac{M_{cr}}{M_{max}}\right)^m\right] \tag{1}$$

where f_1 is the deflection of the uncracked section; f_2 is the deflection of the transformed cracked section; $\beta_1 = 0.5$ is a non-dimensional coefficient accounting for bond properties of FRP bars; β_2 is a non-dimensional coefficient accounting for the duration of loading (1.0 for short time loads, 0.5 for long time or cyclic loads); M_{max} is the maximum moment acting on the examined element; M_{cr} is the cracking moment calculated at the same cross section of M_{max}; m is a bond coefficient that CNR-DT 203 prescribes "*to be set equal to 2, unless specific bond characterization of FRP bars for the investigation of deflection is carried out by the manufacturer, by following the procedure to determine a different value of* m *reported in Appendix E*"; such procedure can be summarized as follows: on the basis of a population of at least five elements of concrete reinforced with FRP bars, that shall be subjected to four-points bending test, deflections are measured for fixed load values, ensuring that for a single test there is a number of at least five acquisitions over time interval between 20% and 60% of the ultimate load, P_{ult}. The same load values are used to calculate the theoretical deflections starting from equation (1). The exponent m is determined on the basis of the comparison between analytical and experimental results, using an appropriate statistical analysis, after assigning the unitary value to β_1

and β_2. A similar approach was used herein to assess the accuracy of value m=2 on the basis of an extensive set of data available in literature.

2 BOND

The modulus of elasticity of glass and aramid FRP bars is about one-fifth that of steel. Even though carbon FRP bars have a higher modulus of elasticity than glass FRP bars, their stiffness is about two-thirds that of steel reinforcing bars. Lower stiffness causes larger deflections and crack widths for FRP reinforced members which can affect serviceability (Toutanji and Saafi, 2000). Since an important role is played by bond between FRP bar and concrete, the bond behavior of FRP reinforced specimens is of interest in this investigation.

Bond between reinforcement and concrete is affected by many factors. The major factors influencing the bond behavior of FRP reinforced concrete are as follows (Pay, 2005):

- *Concrete cover and bar spacing*; an increase of concrete cover and bar spacing enhances the bond capacity, although this aspect is less prominent for larger diameter bars.
- *Concrete compressive strength*. The effect of concrete strength is not fully understood for FRP reinforced specimens, since there is only limited data available for FRP bar reinforced specimens. Nanni et al. (1995) investigated the effect of concrete strength on bond behavior using pullout specimens and found that concrete strength does not have any influence on pullout failures. However Malvar (1994) found that, for splitting failures, an increase in concrete strength results in an increase in bond strength.
- *Development length*; an increase in the development length of a reinforcing bar will increase the total bond force transferred between the concrete and the reinforcement; as for steel, when the bonded length increases, the effectiveness of the bonded length decreases, thus the relative gain with increase in development length reduces. Further study is needed to quantify this effect.
- *Transverse reinforcement*; the presence of transverse reinforcement in the development region prevents the progression of splitting cracks; therefore, the bond force required to cause failure of the bar increases (Orangun et al., 1977, Tepfers, 1982, Darwin et al., 1996 a, b). As the bond strength increases with an increase in transverse reinforcement, eventually the failure mode changes from splitting to pullout. Additional transverse reinforcement above that required to cause a pullout failure is unlikely to increase the anchorage capacity of the section (Orangun et al., 1977).
- *Bar size*; the bar size has a direct influence on the bond strength of FRP reinforced beams. As the bar size increases for a given development and splice length, the total bond force developed by the bar increases. However, the rate of increase in the bond force is lower than the increase in bar area. Consequently, bond stresses are lower for larger diameter bars.
- *Surface deformation of the reinforcement*; the force transfer between FRP bars and concrete is mainly due to chemical adhesion and friction between the concrete and the reinforcement; bearing of concrete on the surface deformation is minimal. Makitani et al. (1993), Malvar (1994), and Nanni et al. (1995) studied the effect of surface deformation on the bond strength of FRP reinforced specimens through pullout tests, concluding that the surface deformation of the bar has an influence on the bond strength.

3 CALIBRATION OF BOND COEFFICIENT *m*

3.1 *Test specimens and variables*

The data set consisted of sixty-seven concrete beam and slab specimens reinforced with continuous FRP bars, tested through four points bending tests and available in literature (Benmokrane et al., 1996, Alsayed, 1998, Masmoudi et al., 1998, Theriault & Benmokrane, 1998, Alsayed et al., 2000, Pecce et al, 2000, Toutanji & Deng, 2003, Yost et al., 2003, El Salakawy & Benmokrane, 2004, Al Sunna et al., 2006, Laoubi et al., 2006, Rafi et al, 2006). Fig. 1 shows the cross section and the test setup layout.

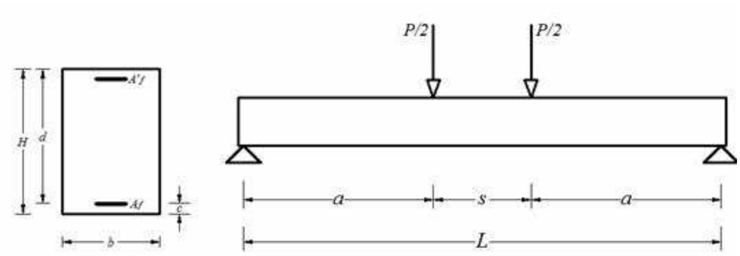


Figure 1. Cross Section and Test Setup Layout

The cross section width, *b*, ranged between 120 and 1000 mm; the height, *H*, ranged between 180 and 550 mm; the length, *L*, varied between 1500 and 3400 mm; the distance between the support and the applied load, *a*, ranged between 500 and 1450 mm; the constant moment zone, *s*, varied between 100 and 1000 mm.

As for the concrete used for casting the specimens, the mean compressive strength, f_c, ranged between 30 and 97 MPa; the mean tensile strength for flexure, $f_{ct,fl}$, ranged between 2.9 and 5.2 MPa; and the compressive modulus of elasticity, E_c, ranged between 23 and 46 GPa; in particular for E_c also the corresponding theoretical values were computed (ranging between 23 and 41 GPa), using the following relationship (ACI 318, 1996):

$$E_{c,the} = 4263 \cdot \sqrt{f_c} \tag{2}$$

The FRP reinforcement included glass (62 specimens) and carbon bars (5 specimens) with different sizes and surface deformations. The bars tensile strength, f_{fu}, varied from 507 to 3912 MPa; the modulus of elasticity, E_f, varied from 36 to 136 GPa; and the diameter of bars in tension, ϕ, ranged between 9 and 22 mm.

All the geometrical characteristics and materials details related to the specimens considered are reported elsewhere (Fico, 2007).

3.2 *Cracking Moment*

In order to calibrate the bond coefficient "*m*" in formula (1), three different cases were analyzed, namely: 1. $M_{cr,exp}$, & $E_{c,exp}$; 2. $M_{cr,the}$, & $E_{c,exp}$; 3. $M_{cr,exp}$, & $E_{c,the}$, where $M_{cr,exp}$ and $M_{cr,the}$ are the experimental and the theoretical value of the cracking moment, respectively. The definition of the cracking moment is important since it influences the evaluation of deflection for FRP reinforced members (Pecce et al., 2001); since $M_{cr,the}$ depends on the concrete strength in tension, that is a very uncertain parameter and usually can not be directly measured, but computed depending on the strength in compression, the introduction of the experimental value of the cracking moment M_{cr} allows to examine the model efficiency disregarding the influence of the uncertainties due to $M_{cr,the}$ (1st case); nevertheless, evaluating $M_{cr,the}$ is significant for the model application (2nd case); similarly, the significance of $E_{c,the}$ instead of $E_{c,exp}$ in the model application was taken into account (3rd case).

All values of the ultimate load, P_{ult}, the moment of inertia of both the un-cracked (I_1) and cracked section (I_1), and of $M_{cr,exp}$ and $M_{cr,the}$ relating to all specimens considered are reported elsewhere (Fico, 2007).

3.3 *Calibration Analysis*

For each of the three cases reported in § 3.2, the calibration of the exponent "*m*" was carried out by computing the standard (e_1) and the mean error (e_2):

$$e_1 = \sqrt{\frac{\sum_{i=1}^{n} \left(\frac{f_{the} - f_{test}}{f_{test}} \right)_i^2}{n}} ; e_2 = \frac{\sum_{i=1}^{n} \left(\frac{f_{the} - f_{test}}{f_{test}} \right)_i}{n} \tag{3}$$

where f_{the} and f_{test} are the theoretical and the experimental value of the deflection, respectively; *i* is the generic test, and *n* is the number of considered points; e_1 can be considered as a measure of the reliability of equation, whereas e_2 is a measure of the safe level of the model (e_2>0: the model is safe). The errors have been calculated in a load range which could be significant of serviceability conditions, namely 20 to 65% of ultimate load, P_{ult}; with load steps of 5%, 10 different deflection values in correspondence of as many load values were measured for each test.

Following a summary of the calibration analysis performed is reported:

- Compute the theoretical deflection corresponding to a percentage value α of the applied load (20%<α<65%), f_{the}^{α};
- Measure the corresponding experimental deflection, f_{test}^{α}, on the plots available in literature (67 out of 180 specimens available in literature could be selected);
- Compute e_1 and e_2.

By varying the bond coefficient m the minimum value of e_1 (with e_2>0) was found for each of the three cases analyzed, as follows.

1. ***M***

Fico (2007) reports the values of theoretical deflections computed for each load step when setting $M_{cr}=M_{cr,exp}$ and $E_c=E_{c,exp}$, according to equation (1), and the corresponding experimental values measured. The bond coefficient corresponding to the minimum value e_1=0.212 is m=0.872. As for e_2, since the value derived is nearly zero (e_2=-0.062), it can be concluded that the analytical model is sufficiently reliable.

A different evaluation was performed deriving m for each load step of every single test, after setting $f_{exp}=f_{the}$, so that $f_{exp}=f_1\cdot\gamma+f_2\cdot(1-\gamma)$. Therefore γ was derived:

$$\gamma=\frac{f_{exp}-f_2}{f_1-f_2}\ (5),\ \text{from which: } m=\log_{\frac{M_{cr}}{M_{max}}}\left(\frac{f_{exp}-f_2}{f_1-f_2}\right)\ (6)$$

Hence the following quantities were plotted as shown in Fig 2-a:

$$\left[\left(\frac{M_{cr,\exp}}{M_a}\right)^{m=\mathrm{var}},\left(\frac{M_{cr,\exp}}{M_a}\right)\right];\left[\left(\frac{M_{cr,\exp}}{M_a}\right)^{0.872},\left(\frac{M_{cr,\exp}}{M_a}\right)\right];\left[\left(\frac{M_{cr,\exp}}{M_a}\right)^{2},\left(\frac{M_{cr,\exp}}{M_a}\right)\right]\ (M_a=\alpha M_{max}).$$

It can be noticed that points corresponding to m=0.872 approximate points with m=var. better than points corresponding to m=2; thus, m=0.872 is suggested in replacement of m=2 in equation (1).

2. **$M_{cr,the}$ & $E_{c,exp}$**

The significance of model was evaluated in the second case computing theoretical deflections for each load step after setting $M_{cr}=M_{cr,the}$ and $E_c=E_{c,exp}$, according to equation (1), and comparing the results with the corresponding experimental values measured. The same procedure already explained for the first case was followed, computing e_1 and e_2 for each specimen tested.

The bond coefficient corresponding to the minimum value e_1=0.318 is m=0.790. With respect to case 1) it can be observed that the average standard error e_1 in case 2) is higher, and that the average mean error, e_2, is considerably lower than zero (i.e. e_2=-0.205), confirming that considering the analytical value of M_{cr} instead of the corresponding experimental value decreases the model reliability.

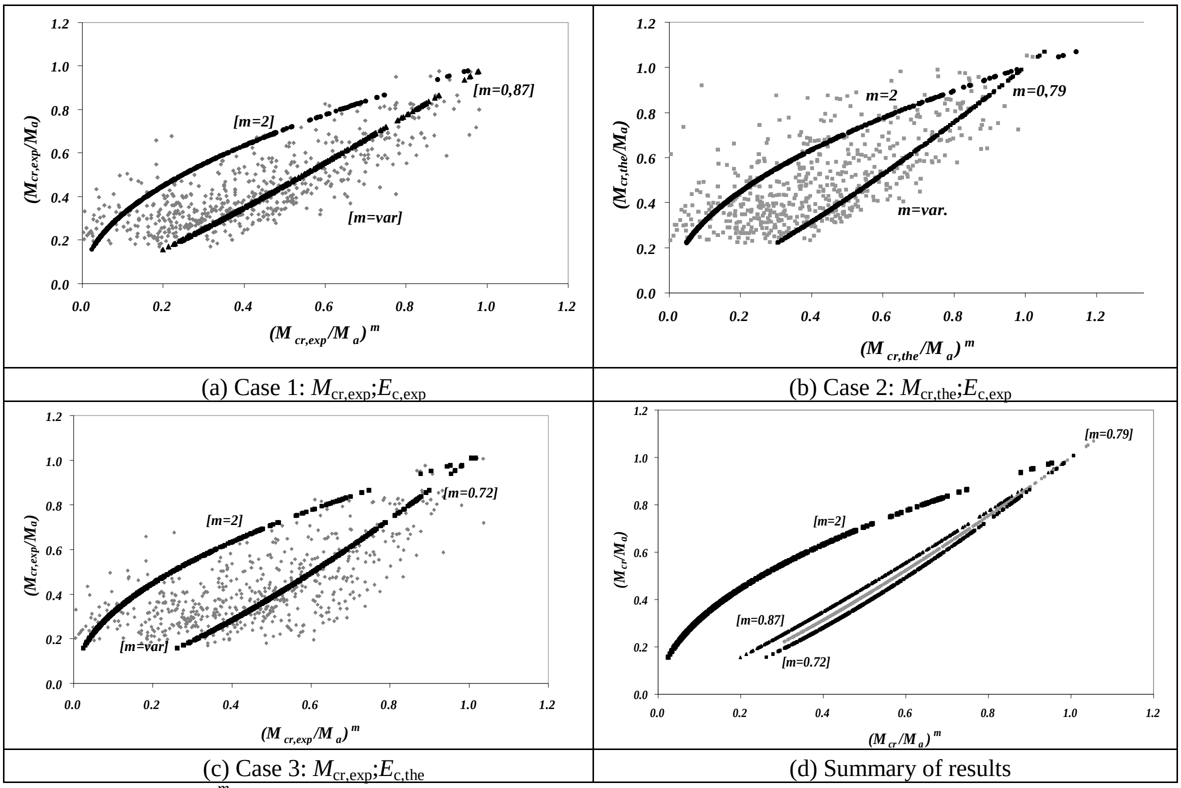
Similarly to case 1, the plot shown in Fig. 2-b was derived. None of the two lines (m=2 and m=0.79) approximates the points with m=var. properly, confirming that the m=2 line is not enough reliable and that considering $M_{cr,the}$ instead of $M_{cr,exp}$ implies an accuracy reduction of the model proposed.

3. **$M_{cr,exp}$ & $E_{c,the}$**

The significance of considering $E_{c,the}$ instead of $E_{c,exp}$ in the model application was taken into account in case 3). The theoretical deflections were computed for each load step after setting $M_{cr}=M_{cr,exp}$ and $E_c=E_{c,the}$, according to equation (1), and comparing the results with the corresponding experimental deflections already measured.

The same procedure already explained for the first two cases was followed, computing e_1 and e_2 for each specimen tested. The bond coefficient corresponding to the minimum value e_1=0.248 is m=0.720. Case 3) can be considered intermediate between cases 1) and 2), its average standard error e_1 being higher than e_1 of case 1), but lower than e_1 of case 3), yet quite reliable as it resulted for case 1) (e_2=-0.059).

As for case 1) and 2), the plot of Fig. 2-c was derived, that confirms the results already discussed: the line corresponding to m=0.79 approximates points with m=var. better than line with m=2, confirming that the m=2 line is not enough reliable.



(a) Case 1: $M_{cr,exp}$;$E_{c,exp}$ (b) Case 2: $M_{cr,the}$;$E_{c,exp}$

(c) Case 3: $M_{cr,exp}$;$E_{c,the}$ (d) Summary of results

Figure 2. $(M_{cr}/M_a)^m$ vs (M_{cr}/M_a)

4 CONCLUSIONS

Fig. 2-d shows the three lines obtained for the three values of m derived, compared to line relating to m=2. It can be observed that the three lines corresponding to the three cases considered are very close and have concave trend, being m<1, converse to the trend of m=2 line. From the comparison of the four lines with respect to the points obtained setting m=var., it can be concluded that the bond coefficient m=2 in equation (1) should be replaced by a value lower than unity. As for the three cases analyzed, Table 1 shows a summary of the results obtained:

Table 1. Summary of results

Case:	e_1	e_2	m
1) $M_{cr,exp}$; $E_{c,exp}$	0,212	-0,062	0,87
2) $M_{cr,the}$; $E_{c,exp}$	0,318	-0,205	0,79
3) $M_{cr,exp}$; $E_{c,the}$	0,248	-0,059	0,72

The first value m_1=0.87 corresponds to the minimum value of the average standard error e_1 with a sufficient level of safety (e_2≈0): this confirms that considering the experimental values of the cracking moment and of the modulus of elasticity of concrete instead of the theoretical values brings to more reliable predictions. Therefore the value m=0.87 to use as bond coefficient when computing deflections of FRP RC elements in equation (1) of CNR-DT 203/2006 is the one proposed. Of the two other cases considered, case 3) where the theoretical value of E_c replaced the experimental value, resulted to give better predictions than case 2), where the theoretical value of M_{cr} was used instead of the corresponding experimental value. The investigation of the available data collected allowed concluding that computing the cracking moment (rather than accounting for its experimental value) penalizes the reliability and the safety of deflection calculations more than considering $E_{c,the}$ instead of $E_{c,exp}$. Nevertheless, the values of m derived in case 2) and in case 3) do not differ from the value of case 1) considerably, with a maximum variation of 17% of m_3 with respect to m_1. Hence, considering the theoretical aforementioned values rather than the corresponding experimental quantities does not penalize the reliability of results considerably.

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